

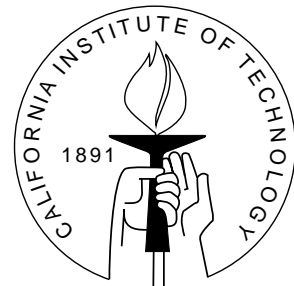
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FLIP-FLOPPING, INTENSE PRIMARIES AND THE SELECTIN OF CANDIDATES

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Abstract

We present a model of two-stage elections in which candidates can choose different platforms in primaries and general elections. Voters do not directly observe the chosen platforms, but rather infer the candidates' ideologies from signals made during the campaign (debates, speeches), where a larger number of signals corresponds to a higher-intensity campaign. This model captures two patterns: (1) the “post-primary moderation effect,” in which candidates pander to the party base during the primary and shift to the center in the general election; and (2) the “divisive-primary effect,” which refers to the detrimental effect of intense primaries on a party's general-election prospects. These effects are obtained in spite of the fact that primary voters are forward-looking and take into account that a more extreme candidate has a smaller chance of winning the general election than a moderate one does.

Key words: primaries, information transmission, strategic voting

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1 Introduction

Political primaries, an influential institution in the American political process, require candidates to obtain a party nomination by vote in order to compete in the general election. Two established facts about primaries are: (1) Candidates tend to pander to the party base during primaries and moderate their platforms after securing the nomination,¹ and (2) Hard-fought primaries can influence a party's chances of winning the election.² The first observation, "post-primary moderation," follows from the premise that primary voters hold more extreme political views than the general-election voters. The second observation, the so-called "divisive primary" hypothesis, suggests that a candidate's prospects in a general election may be affected by the intensity of the primary race.

These two observations seem hardly surprising. Despite this, the theoretical literature lacks a model that can deliver both of these results simultaneously. The reason is

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¹Using U.S. congressional data Burden (2001) shows that candidates adopt more extreme positions in primaries than in general elections.

²The conventional wisdom that hotly contested primaries can damage a party's chances in the general election is based on the theoretical work of Key (1953). Empirical literature that studied this conjecture produced mixed results: Abramowitz (1988), Bernstein (1977) and Lengle-Owen-Sonner (1995) find that intense primaries hurt candidates in the general elections, Alvarez-Canon-Sellers (1995) and Westlye (1991) find that intense primaries help candidates in the general election, Atkinson (1998) and Kenney (1988) find that general election prospects are not affected by the primary intensity, and, finally, Born (1981) and Hogan (2003) find mixed relationship. In this paper, we use theoretical analysis to shed light on the relationship between the intensity of the nomination process and general election outcomes. The mechanism studied here delivers a negative correlation, i.e., it shows that intense primaries are detrimental to a party's chances of winning general elections.

that most existing models use one of two extreme assumptions: either that candidates make binding commitments to electoral platforms (as in Wittman (1983) and Coleman (1972)³), or that announcements made by candidates are purely cheap talk (as in Alesina (1988)). If a candidate commits to a platform, then the mere fact of commitment precludes moderation. If a candidate has no access to a commitment technology, then his general-election prospects should not be affected by the intensity of the primary race. In either case, a model with either of these two assumptions cannot explain both the post-primary moderation and the effects of an intense primary race.

In this paper we develop a model of two-stage elections that captures both the post-primary shift and the divisive-primary effect. In our model, candidates have policy preferences and a *partial* commitment to those policies, which is captured by incorporating costs of lying as well as by having the candidates' platforms revealed imperfectly. Voters are forward-looking and take into account that a more extreme candidate has a smaller chance of winning the general election than a moderate one does. The candidates strategically choose the platforms, depending on which signals are generated and observed by the voters, and candidates' true preferences are partially revealed through signals sent during primary and general-election campaigns. The number of signals serves as a measure of the intensity of the race and determines how much information is transmitted in the two-stage election process.

In equilibrium, candidates “flip-flop” by pandering to the median voter of the primary race during the primary and then shifting to the center once the nomination is obtained. In the primary voters elect a candidate they believe to be more extreme. The extent to which candidates mimic each other depends on the costs of lying and the intensity of each stage. We show that in this equilibrium an increase in the primary intensity lowers the chances of the party holding it to win the general election. This is because intense primaries increase the chances of moderate candidates to lose the nomination and decrease the chances of extreme challengers to win the general election. Finally, we demonstrate that an increase in the primary intensity may be beneficial or detrimental for the welfare of the party holding it depending on the ideology of the incumbent.

The trade-off at the heart of the model is a classic one in political economy: the probability of winning versus the policy outcome should you win. This trade-off is the key idea in the work-horse models of Wittman (1983) and Calvert (1985). The difference in this paper is that this trade-off is being made by the median voter of the primary election rather than the candidate herself. To execute the trade-off, the primary median voter has to learn the type of candidate he is nominating. This selection problem itself induces a trade-off: the primary median voter wants to nominate a more extreme type (which is closer to his policy preferences) but as he learns whether a candidate is extreme or not, so too does the general election median voter. This lowers the probability further

³Wittman (1983) studies one-stage election model with policy-motivated candidates, whereas Coleman (1972) investigates two-stage election model (with primaries and general elections) with office-motivated candidates. Both models assume that at the beginning of the election candidates choose one position, which will be implemented if they get elected.

that an extreme nominee will win the general election both directly and indirectly. The indirect effect is that a candidate that is strongly perceived to be extreme will pander less to the general election median voter.

The rest of the paper is structured as follows. Section 2 lays out the model. In Section 3 we characterize the pandering equilibrium, in Section 4 we obtain comparative statics results, and in Section 5 we discuss welfare implications. In Section 6 we formulate several additional implications of the model and present two extensions: one in which both parties hold primary elections and one in which candidates' types are drawn from the continuous distribution. The related literature is summarized in Section 7, and, finally, in Section 8 we offer some conclusions.

2 Model

We build upon the standard one-dimensional policy location game by Downs (1957). A policy space is the closed interval $P = [-1, 1]$. There is a continuum of voters with Euclidean policy preferences on P . A voter is identified by his ideal point $z_i \in P$. The utility of a voter z_i if policy p is implemented is $u(z_i, p) = -|z_i - p|$. The position of the median voter m^{Pop} is not known with certainty: $m^{\text{Pop}} \sim U[-a, a]$ with $E[m^{\text{Pop}}] = 0$ and $a \in (0, 1)$.

There are two parties: a left-wing party (Democrats) and a right-wing party (Republicans). A member of one party is currently holding the office (incumbent). The incumbent will be challenged by the nominated member of the other party in the general election. The non-incumbent party selects its nominee by conducting a primary election. Without loss of generality, we assume that incumbent belongs to the Republican party; thus, it is the Democratic party that holds a primary election to select its nominee for the general election. Moreover, there are two Democratic party candidates $j = A, B$ who compete in the primary. For a candidate j , winning the office involves the defeat of the other Democrat in the primary and the defeat of the Republican incumbent in the general election. The position of the incumbent, R , is known because he has already served one term prior to the current election.⁴

Each Democratic candidate $j = A, B$ is equally likely to be liberal $t^j = L$ or moderate $t^j = M$. A candidate's ideology (type) determines his sincere political beliefs; it is innate and is known only by the candidate.⁵ We assume that $m^{\text{Dem}} \leq L < M \leq E[m^{\text{Pop}}] \leq R$ where m^{Dem} denotes the ideal point of median Democrat.⁶ The median Democrat has a

⁴In section 6.2 we consider the extension of this model, in which both parties hold primary elections and the ideologies of the candidates of both parties are unknown.

⁵Section 6 presents an extension of this model, in which candidates' types are drawn from a continuous distribution.

⁶The assumption that $m^{\text{Dem}} \leq L$ is not crucial, as a liberal candidate can be more left-wing than the median Democrat. What is important is that a candidate with a liberal ideology is closer to the median Democrat than a candidate with a moderate ideology; that is, $|M - m^{\text{Dem}}| > |L - m^{\text{Dem}}|$.

known position $m^{\text{Dem}} = -\frac{1}{2}$.⁷

During the primary, two Democrats compete by choosing a platform, which represents the probability distribution over positions L and M . Voters do not directly observe the platforms chosen by the candidates, but rather observe m_1 signals randomly drawn from each candidate's platform. Platforms represent how strongly a candidate emphasizes positions L and M during his campaign. The more weight a candidate puts on position L , the more he will stress issues that appeal to the base of the Democratic Party, and a random draw from this platform is more likely to be an L signal. Correspondingly, the more weight a candidate puts on position M , the more he will raise issues that are close to the hearts of moderate voters and hence will sound like a moderate. It is the politician (and his team) that decides which issues to emphasize during main speeches and which ones to put aside.⁸ All voters observe signals from the primary race, but only those that belong to the Democratic party cast their votes. The winner of the primary, determined by the majority of votes cast, will challenge the Republican incumbent in the general election.

During the general election, the challenger chooses a (possibly different) platform and m_2 signals are randomly drawn from this platform and observed by voters. As before, the number of signals m_2 is an exogenously determined parameter which measures the intensity of the general election: a higher m_2 means a higher intensity of the general election. All voters cast a vote in the general election. The winner of the general election, determined by the preferences of the median voter m^{Pop} , implements his preferred policy.

The platform of candidate j with type t^j in the primary race is denoted by $\text{plat}_1^{t^j}$ and the platform of the challenger in the general election is denoted by $\text{plat}_2^{t^j}$. We will use the following shortcut $(1 - w, w)$ to denote the platform with weight $1 - w$ on position L and the weight w on position M .

Candidates are policy-motivated just like voters. A candidate with type t gets utility of $-|t - p|$ if policy p is implemented. In addition, each candidate incurs the cost of misrepresenting his true ideology every time he does so. Specifically, we assume that a candidate j with type $t^j = L$ ($t^j = M$) that runs on a platform $(1 - w, w)$ pays the cost of w^c ($(1 - w)^c$) where $c \in \mathbb{N}$ and $c \geq 2$. So, a candidate who puts all the weight on his true position pays no costs, while a candidate who misrepresents his type completely pays the cost of 1.⁹ Figure 1 summarizes graphically the topography of the electoral

⁷This assumption guarantees that the results of the primary election provide no information about the location of the median voter in the general election.

⁸For instance, in the primary campaign of 2008, President Obama expressed the firm intention of renegotiating NAFTA. However, there was not much talk about that issue during the general election campaign. Why is that? Presumably, the topic of NAFTA regulations is of greater concern to the Democratic base, which is the decisive force in the primary, than to the moderate voters during the general election.

⁹Such costs may arise due to the costly action required by constructing a coherent platform that stresses issues that are not the candidates' priorities and contradict his previous statements (actions). See also Banks (1990) and Calander-Wilkie (2007) who use costs of lying to study electoral competition

game we study in this paper.

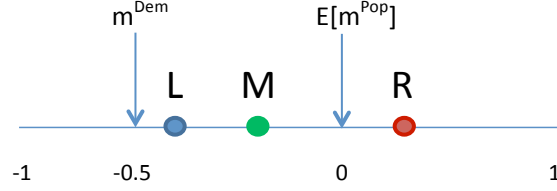


Figure 1: Political Spectrum

Order of moves:

1. *Information stage*

Each Democratic candidate $j = A, B$ privately learns his ideological type.

2. *Primary stage*

- (a) Democratic candidates choose their platforms: $plat_1^{tj}$ for $j = A, B$.
- (b) All voters observe m_1 signals drawn from candidates' platforms and form beliefs about candidates' true ideologies.
- (c) Voters that belong to the Democratic party vote for one of the candidates. The nominee, determined by the preferences of the median Democrat m^{Dem} , continues the race in the next stage and will henceforth be called the challenger.

3. *General election stage*

- (a) The challenger chooses a platform to run on in the general election: $plat_2^{tj}$.
- (b) All voters observe m_2 random signals drawn from the challenger's platform and update their beliefs about his true ideology.
- (c) All voters cast a ballot for either the challenger or the incumbent. The winner is determined by the preferences of the median voter m^{Pop} .

4. *Implementation stage*

The elected official implements his preferred policy, and payoffs are determined.

in one-stage elections. While Banks and Calander-Wilkie assume that only the winning candidate bears the costs of the contradiction between "what candidate said" and "what candidate did", the current paper assumes that these costs incurred by *all* candidates who misrepresent their true ideology, as in an all-pay auction.

This electoral game is characterized by the following set of exogenous parameters: (m_1, m_2) represent the intensities of the primary and the general election stages; c is the parameter of the cost function of the candidates; L , M , and R capture the candidates' spatial locations (ideologies); and, finally, a reflects the uncertainty about the general-election median voter.

Before launching into formal treatment of the model, we note that the concept of intensity of elections has been largely ignored in theoretical models of spatial competition. Empirical studies, however, often control for the intensity of elections. In this paper we take the first step in incorporating the concept of election intensity into the equilibrium model of elections. We treat intensity as an exogenous characteristic of a race. A few examples of campaign aspects that cannot be easily manipulated by the candidates are: the capacity of voters to absorb information, how much time and resources voters decide to devote to a particular race and the media coverage of a race. These are the aspects of a race that we call *intensity of a race*. Our main question is how intensity affects the positioning of candidates in equilibrium.¹⁰

2.1 Equilibrium Concept

To analyze the outcomes of electoral game we will use the standard solution concept of sequential equilibrium developed by Kreps-Wilson (1982). We focus on symmetric sequential equilibria, in which candidates with the same ideology employ the same strategy at each stage of the game. The objects of symmetric sequential equilibrium are: (1) the strategy of each type of candidate in the primary stage $(plat_1^L, plat_1^M)$, (2) the strategy of the challenger in the general election stage $(plat_2^L, plat_2^M)$, (3) the system of beliefs of voters, and (4) the voting behavior at each stage. After every history, the strategies of each type of candidate and of voters are sequentially rational, given the system of beliefs and the beliefs are consistent with the strategy profile. Voters are forward-looking and use Bayes' rule to update their beliefs about candidates' ideologies. A voter that has a strict preference for one of the candidates necessarily votes for him, while a voter who is indifferent randomizes equally between the two candidates. Abstention is not allowed.

3 Pandering equilibrium

In this section we present the main result of the paper: a *pandering equilibrium* (*PE*), in which candidates pander to the median Democrat during primaries and shift to the center during general elections. The driving force behind this result is the need to appeal to populations with different preferences in the primary than in the general election, with the median Democrat located to the left of the median general-election voter.

¹⁰There are, of course, other aspects of a race which are controlled by candidates such as number of TV ads. We leave the question of what happens when the candidates can influence intensity for a future research.

Theorem 1. Consider a two-stage electoral game in which $m^{Dem} = -\frac{1}{2} \leq L < M \leq E[m^{Pop}] = 0 \leq R$ and $-2a - L < R < 2a - M$. If the intensity of a primary is sufficiently low ($m_1 < c$) and uncertainty about the general election is sufficiently high ($a > a^*$), then there exists a pandering equilibrium (PE) of the game which has the following properties. In the primary stage, liberal and moderate candidates play $plat_1^L = (1, 0)$ and $plat_1^M = (y, 1-y)$, respectively, where $y \in (0, 1)$. After observing m_1 messages from the candidates' platforms, voters update their beliefs regarding the candidates' ideologies:

$$p_1^j = Pr[t^j = M] = \begin{cases} \frac{y^{m_1}}{y^{m_1} + 1} & \text{if all messages were L} \\ 1 & \text{if at least one message was M} \end{cases}$$

If both candidate have the same posterior beliefs then each candidate has an equal probability of winning the primary, whereas if $p_1^j \in (0, 1)$ and $p_1^k = 1$, then candidate j wins the primary. In the general election stage, a moderate challenger plays $plat_2^M = (0, 1)$, while the behavior of a liberal challenger depends on the intensity of the general election: if $m_2 < c$ then $plat_2^L = (1 - x, x)$ where $x \in (0, 1)$, otherwise $plat_2^L = (1, 0)$.

There are two conditions that guarantee the existence of the PE. The first one, $m_1 < c$, ensures that a moderate candidate finds it worthwhile to mimic a liberal ideology in the primary by putting some positive weight on the L position. The second condition, $a > a^*$, ensures that the median Democrat supports a more liberal candidate over a less liberal one in the primary election. We now present the main steps of the solution (the detailed proofs are in Appendix A).¹¹

3.1 Behavior of voters in the general election

Preferences of the median voter m^{Pop} determine the winner of the general election. We denote by p_2 the probability that voters believe challenger has type M after observing m_2 messages from his platform and by $f(p_2)$ the probability of such a challenger to beat the incumbent. Using the assumption about uncertain location of m^{Pop} we derive:

$$f(p_2) = f_0 + f_1 \cdot p_2 \text{ where } f_0 = \frac{R + L + 2a}{4a} \text{ and } f_1 = \frac{M - L}{4a}$$

The condition $-2a - L < R < 2a - M$ guarantees that the result of the general election is never a certain event no matter how much information was revealed during the election process. The condition $M > L$ guarantees that the function $f(p_2)$ is strictly increasing in p_2 . That is, a challenger with a higher chance of being a moderate (i.e., the one that is closer to the $E[m^{Pop}]$) has a higher chance of winning the general election.

¹¹The PE is unique if the intensity of the general election is sufficiently high ($m_2 \geq c$). When $m_2 < c$, however, both types have an incentive to keep their identities hidden in the primary: the moderate candidate does so because he wants to increase his chance of winning the primary; the liberal candidate does so in order to enjoy a higher chance of winning the general election in the event that he wins the primary and then successfully mimics a moderate candidate in the general election. Thus, for some parameters of the game there exists an equilibrium in which both types mix in the primary.

The uncertainty parameter a determines the slope of $f(p_2)$: as a increases, the slope decreases.¹² Finally, the ideology of the incumbent R shifts the function $f(p_2)$ up and down: the more conservative R is, the higher the chance that a liberal challenger that has revealed his type wins the general election.

3.2 Behavior of candidates in the general election

The behavior of a challenger in the general election depends on the voters' beliefs about his type after the primary campaign.

Lemma 1. *If voters are certain about challenger's type after the primary campaign or if the intensity of the general election is sufficiently high ($m_2 \geq c$) then the unique continuation strategy of a challenger consistent with the equilibrium is $plat_2^M = (0, 1)$ and $plat_2^L = (1, 0)$. If voters are uncertain about challenger's type, $p_1 = Pr[t^{challenger} = M] \in (0, 1)$ and the intensity of the general election is sufficiently low ($m_2 < c$), then the unique continuation strategy of a challenger consistent with the equilibrium is $plat_2^M = (0, 1)$ and $plat_2^L = (1 - x, x)$ where x is determined by*

$$x^{c-m_2} = \frac{m_2}{c} \cdot \frac{(R-L)(M-L)}{4a} \cdot \frac{p_1}{p_1 + (1-p_1)x^{m_2}} \quad (1)$$

Corollary 1. *If $m_2 < c$ then $x(p_1)$ is an increasing function of p_1 .*

That is, a "shift to the center" by a liberal challenger in the general election is bigger when voters believe that he is more likely to actually be a moderate. To intuit this result, consider a liberal challenger who won the primary with a very small p_1 . In this case, voters are fairly confident that the challenger is a liberal. This challenger will have a hard time convincing voters that he is actually a moderate. Given that misrepresentation is costly, such a challenger will pander less towards the position of the general election median voter. This result highlights the danger of an early resolution of uncertainty in the primary. After an intense primary, a liberal challenger panders less to the general election median and as a result wins less often.

3.3 Behavior of voters in the primary election

Lemma 2. *If $a > -\frac{M+L}{2}$ then if $p_1^j \in (0, 1)$ and $p_1^k = 1$ then candidate j wins the primary.*

¹²If the location of a median voter in general election is relatively uncertain, liberal and moderate challengers have similar chances of winning the office. On the other hand, if the location of a median voter is more or less known, a small increase in the degree to which voters believe that the challenger has a moderate ideology (p_2) makes a significant difference in terms of his probability of winning.

Put in words, the more liberal candidate wins the primary. The main trade-off that voters face in a two-stage election is the need to weigh two factors: what they believe the candidate's ideology is, and his chances of winning the general election. When uncertainty about m^{Pop} is sufficiently high, the second consideration becomes less important, because the probability of winning the general election function $f(p_2)$ is relatively flat.

3.4 Behavior of candidates in the primary stage

Lemma 3. *If $a > a^*$ and $m_1 < c$, then there exists a unique $y \in (0, 1)$ such that $\text{plat}_1^L = (1, 0)$, and $\text{plat}_1^M = (y, 1 - y)$ is the optimal behavior of candidates in the primary,¹³ where*

$$a^* = \begin{cases} \frac{R-M}{2} - \frac{M-L}{2} & \text{if } R > M - 2L \\ -\frac{M+L}{2} & \text{if } R \leq M - 2L \end{cases} \quad (2)$$

A sufficiently low intensity in the primary is a necessary condition for pandering in the primary: if, on the contrary, $m_1 \geq c$ then a moderate candidate prefers to reveal his true type during the primary campaign. As we have shown in section 3.2, the behavior of a liberal challenger in the general election depends on the intensity of the general election. Therefore, we distinguish two cases:

- When the intensity of the general election is sufficiently high ($m_2 \geq c$), there exists a *PE* such that, in the primary stage, candidates play $\text{plat}_1^L = (1, 0)$ and $\text{plat}_1^M = (y, 1 - y)$ where

$$\frac{c}{m_1} y^{c-m_1} = \frac{f_0(M-L) + f_1(R-M)}{4} \quad (3)$$

and in the general election both types of a challenger separate.

- When the intensity of the general election stage is sufficiently low ($m_2 < c$), there exists a *PE* such that, in the primary stage, candidates play $\text{plat}_1^L = (1, 0)$ and $\text{plat}_1^M = (y, 1 - y)$, and in the general election stage a challenger plays $\text{plat}_2^M = (0, 1)$ and $\text{plat}_2^L = (1 - x, x)$, where $(x, y) \in (0, 1) \times (0, 1)$ are determined by the system below:

$$\begin{cases} \frac{c}{m_1} y^{c-m_1} = \frac{f_0(M-L)}{4} + \frac{f_1}{4} \cdot \frac{(R-M)(y^{m_1} - 2x^{m_2}) + (R-L)x^{m_2}y^{m_1}}{x^{m_2} + y^{m_1}} \\ \frac{c}{m_2} x^{c-m_2} = \frac{(R-L)(M-L)}{4a} \cdot \frac{y^{m_1}}{y^{m_1} + x^{m_2}} \end{cases} \quad (4)$$

To summarize, the behavior of candidates in the *PE* is consistent with the *post-primary moderation effect*: candidates cater to the median of the party in the primary campaign and once the nomination is secured, they moderate their platforms during the general election.

¹³Notice that $a^* > -\frac{1}{2}$, which means that if we restrict the values of the uncertainty parameter a to be in the region $a \in (-\frac{1}{2}, a^*)$ then naturally general-election median is always located to the right of the democratic median.

4 Comparative statics

In this section we study how the behavior of candidates in the *PE* changes with changes in primary and general-election intensities.

Theorem 2. *Consider $m_2 < c$. If $m_2 \uparrow$ then $y \uparrow$ and $x^{m_2} \downarrow$.*

Higher intensity in the general election race has two effects: (1) a moderate candidate is more willing to lie in the primary campaign and (2) a liberal challenger is more often revealed to be a liberal in the general election stage. The intuition behind these effects is as follows. Since voters observe more signals in a high-intensity general election, a liberal challenger is likely to send at least one *L* signal, revealing that he is a liberal. A moderate challenger, who never sends an *L* message, therefore has a better chance of winning a high-intensity general election. Anticipating the higher likelihood of winning the general election, a moderate candidate will be willing to mimic liberal behavior to a higher extent and incur more costs at the primary stage to win nomination. For $m_2 \geq c$, both types of a challenger separate in the general election, so a change in m_2 does not impact the primary-stage behavior in the *PE*.

Theorem 3. *Consider $m_1 < c$. If $c \leq m_2$ then $m_1 \uparrow \Rightarrow y^{m_1} \downarrow$. If $m_2 < c$ then $m_1 \uparrow \Rightarrow y^{m_1} \downarrow$ and also $x \downarrow$.*

The higher the intensity of the primary, the harder it is for a moderate candidate to mimic a liberal during that primary. As the number of signals increases, it becomes more likely that an *M* message will emerge and reveal the moderate's true ideology to primary voters. Moreover, after a high-intensity primary, a liberal challenger will engage in less mimicry of a moderate ideology in the general-election stage (see Corollary 1).

Corollary 2. “Divisive Primary Effect.” *Intense primaries decrease the chances of Democrats to win the election.*

A negative relationship between an intense primary and the chances of Democratic candidates to win a general election follows directly from Theorem 3 and Corollary 1. Intense primaries decrease the chances of a moderate Democrat to win the nomination as well as the chances of a liberal challenger to win the general election. Both effects are detrimental to the Democratic party.

5 Welfare of the Democrats

The intensity of the primary race affects not only the chances of Democrats to win the election but also the welfare of Democrats. To define the welfare of Democrats, we need to specify the distribution of ideal points of voters that belong to the Democratic Party. To make things simple, we assume that registered Democrats have ideal points distributed uniformly over the interval of $[-1, 0]$, which is consistent with the position of median

Democrat we assumed so far, $m^{\text{Dem}} = -\frac{1}{2}$. We denote by $W^{\text{Dem}}(p)$ the welfare of the Democratic Party when policy p is implemented and by $E[W^{\text{Dem}}]$ the overall welfare of Democrats where the expectation is taken over the implemented policies. Then

$$E[W^{\text{Dem}}] = \Pr[L \text{ wins}] \cdot W^{\text{Dem}}(L) + \Pr[M \text{ wins}] \cdot W^{\text{Dem}}(M) + \Pr[R \text{ wins}] \cdot W^{\text{Dem}}(R)$$

$$W^{\text{Dem}}(p) = - \int_{-1}^0 |z_i - p| dz_i \text{ for } p \in \{L, M, R\}$$

The welfare of Democrats is the highest when a liberal candidate wins the election and the lowest when an incumbent wins: $W^{\text{Dem}}(L) > W^{\text{Dem}}(M) \geq W^{\text{Dem}}(R)$, where the last equality holds only if $M = R = E[m^{\text{Pop}}]$. Theorem 4 shows that the expected welfare of Democrats can increase or decrease with primary intensity depending on the ideology of the incumbent.

Theorem 4. *Consider $m_2 \geq c$. Then,*

$$m_1 \uparrow \Rightarrow \begin{cases} E[W^{\text{Dem}}] \uparrow & \text{if and only if } R < R^* \\ E[W^{\text{Dem}}] \downarrow & \text{if and only if } R > R^* \end{cases}$$

where $R^* = \min\{0, \frac{-L^2 - 2a(1+L+M) - L(1+M) - M(1+M)}{L+M}\}$.

The welfare of Democrats is determined by a balance of two countervailing effects. On the one hand, intense primaries increase the probability that incumbents win the general election, which is naturally to the detriment of Democrats. On the other hand, liberal candidates tend to win intense primaries more often, which is the best possible outcome for Democrats. When the incumbent is relatively moderate ($R < R^*$), the latter positive effect outweighs the former negative one. When the incumbent is very conservative ($R > R^*$), however, the negative effect dominates the positive one and Democrats suffer from an intense primary.

6 Discussion and Extensions

6.1 Further implications of the model

Flip-flopping behavior of candidates in two-stage elections implies that for a given candidate there is a negative correlation between his chances of winning the primary and his chances of winning the general election. Maisel and Stone (1998) use the data from the Potential Candidate Survey to show that prospective politicians who consider running for the House are aware of this relation (Table 1). 56% of candidates who believe to have a high chance of winning the general election estimate their chances of winning the nomination as unlikely, compared to 26% of candidates who have a low chances in the general election. Similarly, 50% of the candidates who are pessimistic about the general election believe they are likely to win nomination compared to 24% of candidates who are optimistic about the general election stage.

	Prob to win the general election	
Prob to win the primary	Low	High
Unlikely	26%	56%
Toss-up	24%	21%
Likely	50%	24%
# of obs.	108 obs.	102 obs.

Table 1: Beliefs of the Potential Candidates (Maisel-Stone (1998)).

The second implication of the model relates the intensity of the primary and general election stages to the selection of candidates. Controlling for the intensity of the general election, we expect districts with an intense nomination process to elect more extreme legislators on average. Further, controlling for the intensity of the primary races, we expect districts with an intense general election stage to elect legislators who are more moderate on average. These two hypotheses are straightforward implications of the comparative statics results discussed in section 4.

6.2 Both parties hold primary elections

In this section we extend the model to allow both parties to hold primaries and show that the post-primary moderation and intense primary effects hold true in this case as well. To focus attention on the two primaries, we consider the extension of the basic model with a relatively intense general election ($m_2 \geq c$) and symmetric ideological types of Democratic and Republican candidates around the expected median general-election voter. We allow the two primaries to have different intensities (m_1^D, m_1^R) and characterize the unique equilibrium of this game.

Theorem 5. *If $a > \bar{a}$ and $m_2 > c > \max\{m_1^D, m_1^R\}$ then there exists a unique equilibrium of the two-stage election game with both parties holding simultaneous primaries, in which in the primary candidates with extreme ideological types play the truth and moderate candidates mimic them partially. The voters elect a more extreme candidate in both primaries. All types separate in the general election race.*

Corollary 3. *Probability that Republicans (Democrats) win election decreases with the intensity of the Republican (Democratic) primary.*

Theorem 5 and Corollary 3 demonstrate the robustness of the main results of the model: moderate candidates cater to their party’s base during the primary only to move to the center once they secure the nomination and an intense primary hurts the chances of the party engaged in it to win the election, which in turn helps the opposing party.

6.3 Candidates with Continuous Policy Types

In this section we present an extension of the model with a continuous policy space and show that the movement in platforms between the primary and general election persist. We consider the following modified version of the electoral game. A policy space is the closed interval $P = [-1, 1]$. There is a continuum of voters with Euclidean policy preferences on P . A voter with ideal point z_i obtains utility $u(z_i, p) = -|z_i - p|$ if policy p is implemented. The location of the median voter is uncertain:

$$m^{\text{Pop}} \sim U[-a, a] \text{ where } a > 0 \Rightarrow E[m^{\text{Pop}}] = 0$$

The election consists of two stages: the Democratic primary that determines the Democratic nominee and the general election stage, in which the nominee challenges the Republican incumbent. Voters with ideal points $z_i \in [-1, 0]$ belong to the Democratic Party and, thus, participate both in the primary and in the general election. Voters with ideal points $z_i \in (0, 1]$ belong to the Republican Party and vote only once in the general election stage. However, they observe what happens in the Democratic primary and, thus, form the same beliefs about candidates as do the voters from the Democratic Party. The median Democrat is located at $m^{\text{Dem}} = -\frac{1}{2}$.

There are two candidates that belong to the Democratic Party, A and B, who compete in the primary election for the right to proceed to the general election. Each candidate j has a "type" t^j which represents a policy position that he/she will implement if elected. The type of the Republican incumbent is known: $R \in [0, 1]$. The types of Democratic candidates are assumed to be independent and identically distributed random variables according to the continuous uniform distribution

$$\text{for } j \in \{A, B\} \Rightarrow t^j \sim U[m^{\text{Dem}}, Em^{\text{Pop}}]$$

In the primary election, each candidate j sends a message s^j , which is publicly observed by all voters: $s^j : [m^{\text{Dem}}, Em^{\text{Pop}}] \rightarrow [m^{\text{Dem}}, Em^{\text{Pop}}]$. Based on the observed messages, the voters update their priors about candidates' types and the voters that belong to the Democratic Party vote for one of the candidates. The winner, determined by the majority of votes, proceeds to the general election stage, in which the true type of a challenger is revealed and he faces the incumbent with type R . This is an analog of the "intense" general election stage as we refer to it in the base model. Given the assumptions about the uncertainty of the m^{Pop} , a challenger with type t wins general election with probability

$$f(t) = f_0 + f_1 \cdot t = \frac{R + 2a}{4a} + \frac{1}{4a} \cdot t \text{ where } \begin{cases} f(-\frac{1}{2}) > 0 \\ f(0) < 1 \end{cases}$$

Candidates care about winning the elections.¹⁴ A candidate who wins the election gets utility normalized to 1. In addition, if a candidate lies in the primary stage he incurs

¹⁴The version of the electoral game with office-motivated candidates and binary space is very similar to the model studied in this paper. We omit this extension for brevity (available from the author upon request).

costs of lying. The costs of lying are parameterized as follows: a candidate with true type t^j who sends message m during the primary stage pays the cost of $c \cdot |t^j - m|$ where $0 < c < \bar{c}$. The upper bound $\bar{c} = 2 \cdot f\left(-\frac{1}{2}\right)$ ensures that some types of candidates will pool with the other types during the primary. If $c \geq \bar{c}$ then the only equilibrium in this game is the one in which all types reveal their true identity in the primary election (situation similar to $m_1 \geq c$ in the base model).

The expected utility of a candidate j with type t^j that announced m in the primary is

$$EU(t^j)|_m = \lambda(t^j) \cdot f(t^j) - c \cdot |t^j - m|$$

where $\lambda(t^j)$ denotes the probability of type t^j to win the primary race.

Theorem 6. *If the uncertainty about m^{Pop} is sufficiently high ($a \geq \frac{1}{6}$) and the cost of lying is sufficiently low ($c < f_0$) then there exists an uninformative symmetric sequential equilibrium of the electoral game, in which all types of candidates announce $m = m^{Dem}$ in the primary. If voters observe message $m = m^{Dem}$ they do not update their prior belief regarding candidate's type, however, if voters observe out-of-equilibrium message $m' \in (-\frac{1}{2}, 0]$ then they believe it was sent by candidate with type $t^j = 0$.*

Theorem 7. *If the uncertainty about m^{Pop} is sufficiently high ($a \geq \frac{1}{2}$) and the costs of lying are not too high ($f(0) < c < 2f\left(-\frac{1}{2}\right)$) then there exists a partially informative symmetric sequential equilibrium of the electoral game, in which the candidates with types $t^j \in [-\frac{1}{2}, p]$ pool together by announcing $m = m^{Dem}$ and the candidates with types $t^j \in (p, 0]$ separate and play $s(t^j) \leq t^j$, where*

$$p = \frac{1 - 2R - 4a + \sqrt{(4a + 2R - 1)^2 - 8(4ac - R - 2a)}}{4} \text{ and } s(t^j) = \frac{2t^j}{c} \cdot f(t^j)$$

If voters observe $m = -\frac{1}{2}$ then they believe this message comes from a candidate with type $t \in [-\frac{1}{2}, p]$, if voters observe $m = s(t)$ for $t \in (p, 0]$ they infer the precise type of a candidate and if voters observe out-of-equilibrium message $m \in (-\frac{1}{2}, s(p))$ then they believe this message was sent by the candidate with type p . A candidate whose expected type is closer to the ideal policy of m^{Dem} wins the nomination.

7 Related literature

The model presented here belongs to the literature that studies information transmission through electoral competition. The first such model is Banks (1990) who showed that if costs of lying are above critical value, then in equilibrium extreme candidates are willing to reveal their true type, while moderate candidates pool together. Callander-Wilkie (2007) extend Banks's model to allow candidates to have heterogeneous costs of lying and find that, although liars are favored in the elections, the honest types are not always defeated. Kartik-McAfee (2007) study a related situation, in which a fraction of candidates have a "character" and are exogenously committed to a campaign platform.

Finally, Bernhardt-Ingberman (1985) model costly movements of candidates by assuming that candidates are tied to their reputations. Ours is also a signaling model. However, we depart from the above models in that we study two-stage elections, in which candidates face electorates with different preferences in the primary and in the general election. This crucial difference raises the natural question of how much information about candidates' true ideologies is revealed in two-stage elections.

There are several papers that study how primary races affect the selection of candidates. Coleman (1972) and Owen-Grofman (2006) discuss the polarizing effect of primary elections when candidates are constrained to offer the same ideological position in the general election as they have in the primary. Callander (2007) investigates momentum in voting behavior and the emergence of bandwagons. Adams-Merrill (2008) demonstrate that candidates who have stronger campaign abilities are elected in primaries. Alesina-Holden (2008) and Meirowitz (2005) emphasize the advantage of remaining ambiguous in the primary election. In particular, in Meirowitz model candidates benefit from remaining ambiguous about their policy platform during the primary election because the primary reveals information about preferences of the voters. In the equilibrium, candidates converge to the center between the primary and the general election, although the convergence is in the sense that they are ambiguous in the primary and precise in the general election. Hirano-Snyder-Ting (2009) consider a model of distributive politics and show that when the nominee of the party is elected through a primary election, core voters receive positive transfers, whereas they receive nothing when only the general election matters. The flavor of this result is similar to the equilibrium strategy of our candidates in the primary election. However, our paper differs from Hirano et al. in many aspects, including the main focus: we investigate information transmission and the selection of candidates in two-stage elections with policy-motivated candidates, whereas Hirano et al. study the effect of primary elections on the distribution of public resources with office-motivated candidates.

In the recent paper Hummel (2010) presents formal model of two-stage election, in which voters dislike when candidates change their positions between primaries and general elections. One of the results of Hummel paper is similar to the one obtained here: there exists an equilibrium in which candidates choose more extreme positions in the primary and move towards center in the general election, where the extent to which candidates moderate their position depends on the costs of flip-flopping. However, there are several important differences between current paper and that of Hummel both in terms of model specifications and, more importantly, obtained results. First, in Hummel as in many other models of primaries, primaries turn out to be uninteresting in the sense that in the equilibrium, symmetric ex-ante primary candidates are indistinguishable (all liberal candidates choose the same position in the liberal primaries, while all conservative candidates choose the same position in the conservative primary). In our model, candidate are symmetric as well; ex-ante each candidate is equally likely to be liberal or moderate. In spite of that, primaries are full of action, as different sequences of signals are observed from different candidates during the primary campaign. Second, the model developed in this paper provides a unified explanation for both flip-flopping behavior

of candidates and divisive-primary effect and shows that both effects originate from the similar trade-offs.

8 Conclusions

In this paper we develop a signaling model of two-stage elections in which candidates must obtain their party's nomination before competing in the general election. Candidates are policy-motivated and can choose different campaign platforms in every stage of the election. A candidate who misrepresents his true type incurs costs (of lying). We allow different stages of the election to have different intensities, measured by the number of signals observed by voters from the candidates' platforms, and demonstrate that intensities play an important role in the selection process of candidates.

This model provides a unified framework that allows us to examine two commonly observed patterns about primaries: (1) the "post-primary moderation effect," in which candidates pander to the party base during the primary and shift to the center once the nomination is secured and (2) the "divisive-primary effect," which refers to the detrimental effect of intense primaries on a party's general-election prospects.

We finish by noting that the timing of information revelation is important in two-stage elections, as it affects who gets elected, which policies are implemented, and the welfare of the voters. For example, intense primaries might be dangerous for the party in the sense that they reveal too much information about their candidates too early, and this then hurts the party's chances of winning general elections. Depending on the incumbent's ideology, intense primaries may or may not be beneficial for the welfare of the party, since intense primaries filter out moderate candidates during the nomination process.

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9 Appendix

9.1 Characterization of Pandering Equilibrium

Proof of Lemma 1

If $m_2 \geq c$ then both types are happy to separate in the general election. Consider $m_2 < c$. The only interesting case to consider is $p_1 \in (0, 1)$. First we show that if $plat_2^M = (0, 1)$ and $plat_2^L = (1 - x, x)$ where x is as specified in (1) then no type wants to deviate. Assume both types of a challenger follow this strategy. If voters observe only M messages in the general election then they update challenger's type as $p_2 = Pr[t^{\text{challenger}} = M] = \frac{p_1}{p_1 + (1 - p_1)x^{m_2}}$ and if at least one message is L then $p_2 = Pr[t^{\text{challenger}} = M] = 0$. A moderate challenger is clearly happy to play $plat_2^M = (0, 1)$ because $f(p_2)$ is increasing in p_2 . To find optimal platform of liberal challenger assume that instead of playing $plat_2^L = (1 - x, x)$ he plays $plat_2^L = (1 - w, w)$ and characterize the best response function $w^*(x)$ for every $x \in (0, 1)$:

$$w^*(x) = \left((R - L) \frac{m_2 f_1 p_2}{c} \right)^{\frac{1}{c - m_2}}$$

Function $w^*(x)$ is strictly decreasing, continuous and always positive on $x \in (0, 1)$ with $\lim_{x \rightarrow 0} w^*(x) > \lim_{x \rightarrow 1} w^*(x)$. Thus, there exists a unique $x \in (0, 1)$ such that $w^*(x) = x$ which satisfies (1). It is easy to check that L type prefer to play this strategy rather than $plat_2^L = (1, 0)$.

To show that full separation cannot be equilibrium in the general election stage, assume, by contradiction, that it is and consider expected utility of a liberal challenger that plays $plat_2^L = (1 - w, w)$

$$\begin{aligned} EU_{(1-w, w)}^{(L, L)}|_{p_1 \in (0, 1)} &= -w^c - (R - L)[1 - f_0] + (R - L)[f_1 w^{m_2} + p_2 f_1 (1 - w^{m_2} - (1 - w)^{m_2})] \\ \frac{\partial}{\partial w} &= -c w^{c-1} + m_2 w^{m_2-1} \cdot f_1 (R - L) + (R - L) f_1 p_2 \cdot (-m_2 w^{m_2-1} + m_2 (1 - w)^{m_2-1}) \end{aligned}$$

where voters assign belief p_2 to a challenger that sends a mixture of signals M and L. To support separation, it must be that $\frac{\partial}{\partial w} \leq 0$ for all $w \in (0, 1)$. If separation cannot be supported for $p_2 = Pr[t^{\text{challenger}} = M] = 1$ then it can't for any $p_2 < 1$:

$$\frac{\partial}{\partial w} = -c w^{c-1} + m_2 (1 - w)^{m_2-1} \cdot (R - L) f_1$$

Function $\frac{\partial}{\partial w}$ is continuous in $w \in [0, 1]$, $\lim_{w \rightarrow 0} \frac{\partial}{\partial w} = f_1 (R - L) > 0$ and $\lim_{w \rightarrow 1} \frac{\partial}{\partial w} = -c < 0$. Thus, $\exists \bar{w} \in (0, 1)$ such that $\frac{\partial}{\partial w}|_{\bar{w}} > 0$, which shows that separation in the

general election is not part of the equilibrium strategy for $m_2 < c$. It is straightforward to check that any other configuration of the strategies is not part of the equilibrium in the general election stage, **q.e.d.**

Proof of Corollary 1

Use Implicit Function Theorem to determine the sign of $\frac{\partial x(p_1)}{\partial p_1}$:

$$F(x, p_1) = -x + \left(\frac{m_2(R-L)f_1p_1}{c(p_1 + (1-p_1)x^{m_2})} \right)^{\frac{1}{c-m_2}}$$

$$\frac{\partial F(x, p_1)}{\partial x} = -1 - \frac{1}{c-m_2} \left(\frac{m_2(R-L)f_1p_1}{c(p_1 + (1-p_1)x^{m_2})} \right)^{\frac{1}{c-m_2}-1} \cdot \frac{m_2(R-L)f_1p_1}{c} \cdot \frac{(1-p_1)m_2x^{m_2-1}}{(p_1 + (1-p_1)x^{m_2})^2} < 0$$

$$\frac{\partial F(x, p_1)}{\partial p_1} = \frac{1}{c-m_2} \left(\frac{m_2(R-L)f_1p_1}{c(p_1 + (1-p_1)x^{m_2})} \right)^{\frac{1}{c-m_2}-1} \cdot \frac{m_2(R-L)f_1}{c} \cdot \frac{x^{m_2}}{(p_1 + (1-p_1)x^{m_2})^2} > 0$$

$$\frac{\partial x(p_1)}{\partial p_1} = -\frac{\frac{\partial F(x, p_1)}{\partial p_1}}{\frac{\partial F(x, p_1)}{\partial x}} > 0$$

q.e.d.

Proof of Lemma 2

Denote by $Eu(z_i, p_1^j)$ expected utility of voter z_i if candidate j with posterior belief $p_1^j = Pr[t^j = M]$ wins the nomination and behaves optimally in the general election stage. Then

$$\begin{aligned} \text{if } p_1^j &= 1 \text{ then } Eu(z_i, 1) = (f_0 + f_1)(-|z_i - M|) + (1 - f_0 - f_1)(-|z_i - R|) \\ \text{if } p_1^j &= 0 \text{ then } Eu(z_i, 0) = f_0(-|z_i - L|) + (1 - f_0)(-|z_i - R|) \\ \text{if } p_1^j &\in (0, 1) \text{ and } m_2 \geq c \text{ then } Eu(z_i, p_1^j) = (1 - p_1^j) \cdot Eu(z_i, 0) + p_1^j \cdot Eu(z_i, 1) \\ \text{if } p_1^j &\in (0, 1) \text{ and } m_2 < c \text{ then } Eu(z_i, p_1^j) = (1 - p_1^j)(1 - x^{m_2}) \cdot Eu(z_i, 0) + (p_1^j + (1 - p_1^j)x^{m_2}) \cdot Eu(z_i, 1) \end{aligned}$$

where

$$Eu(z_i, p_2^j) = (f_0 + f_1p_2^j)(-|z_i - p_2^jM - (1 - p_2^j)L|) + (1 - f_0 - f_1p_2^j)(-|z_i - R|)$$

$$\text{and } p_2^j = \frac{p_1^j}{p_1^j + (1 - p_1^j)x^{m_2}}$$

Consider two candidates with posterior beliefs $p_1^j = \Pr[t^j = M] \in (0, 1)$ and $p_1^k = \Pr[t^k = M] = 1$. Then all voters with $z_i \in [-1, -\frac{1}{2}]$ support candidate j if and only if $Eu(z_i, p_1^j) > Eu(z_i, p_1^k)$. Using algebraic manipulations, it can be shown that if $a > -\frac{M+L}{2}$ then the last inequality holds true, **q.e.d.**

Proof of Lemma 3.

Assume there exists a **PE** of the dynamic election game, in which candidates play $plat_1^L = (1, 0)$ and $plat_1^M = (y, 1-y)$. Say that candidate A plays equilibrium strategies. Consider what candidate B wants to play. The expected utility of liberal candidate B when he puts weight $1-z$ on position M is

$$-(1-z)^c + EU_{p_1 \in (0,1)}^{(L,L)} \cdot \frac{2 + 2z^{m_1} - z^{m_1}y^{m_1}}{4} + EU_{p_1 \in (0,1)}^{(L,M)} \cdot \frac{2y^{m_1} - z^{m_1}y^{m_1}}{4} + EU_{p_1=1}^{(L,M)} \cdot \frac{(1-y^{m_1})(1-z^{m_1})}{2}$$

where $EU_{p_1^j}^{(k,j)}$ denotes the expected utility of type k when type j wins the nomination and behaves optimally in the general election stage for a given belief $p_1^j = \Pr[t^j = M]$ of the voters after the primary.

Liberal candidate B will play $plat_1^L = (1, 0)$ if and only if $\frac{\partial}{\partial z} > 0$ for all $z \in [0, 1]$:

$$\frac{\partial}{\partial z} = c(1-z)^{c-1} + m_1 z^{m_1-1} \cdot \left[\frac{2-y^{m_1}}{4} EU_{p_1 \in (0,1)}^{(L,L)} - \frac{y^{m_1}}{4} EU_{p_1 \in (0,1)}^{(L,M)} - \frac{1-y^{m_1}}{2} EU_{p_1=1}^{(L,M)} \right]$$

where

$$\begin{aligned} EU_{p_1 \in (0,1)}^{(L,L)} &= -(R-L)(1-f_0-f_1p_2x^{m_2}) \\ EU_{p_1 \in (0,1)}^{(L,M)} &= -(R-L)(1-f_0-f_1p_2) - (M-L)(f_0+f_1p_2) \\ EU_{p_1=1}^{(L,M)} &= -(R-L)(1-f_0-f_1) - (M-L)(f_0+f_1) \end{aligned}$$

Condition $a > -\frac{M+L}{2}$ guarantees that $EU_{p_1 \in (0,1)}^{(L,L)} > EU_{p_1=1}^{(L,M)} > EU_{p_1 \in (0,1)}^{(L,M)} \Rightarrow \frac{\partial}{\partial z} > 0$ for all $z \in [0, 1]$.

Consider now moderate candidate B . His expected utility from playing $plat_1^{t^B=L} = (w, 1-w)$ is

$$-w^c + \frac{(1-w^{m_1})(1-y^{m_1})}{2} \cdot EU_{p_1=1}^{(M,M)} + \frac{2-w^{m_1}}{4} \cdot EU_{p_1 \in (0,1)}^{(M,L)} + \frac{3w^{m_1} + 2y^{m_1} - 2y^{m_1}w^{m_1}}{4} \cdot EU_{p_1 \in (0,1)}^{(M,M)}$$

where

$$\begin{aligned} EU_{p_1 \in (0,1)}^{(M,L)} &= -(R-M)(1-f_0-x^{m_2}f_1p_2) - (M-L)(f_0+x^{m_2}f_1p_2) \\ EU_{p_1 \in (0,1)}^{(M,M)} &= -(R-M)(1-f_0-f_1p_2) \\ EU_{p_1=1}^{(M,M)} &= -(R-M)(1-f_0-f_1) \end{aligned}$$

The best response of $t^B = M$ when $t^A = M$ plays $plat_1^{t^A=M} = (y, 1 - y)$ is

$$w^*(y) = \left[\frac{m_1}{c} \left(\frac{3 - 2y^{m_1}}{4} EU_{p_1 \in (0,1)}^{(M,M)} - \frac{1 - y^{m_1}}{2} EU_{p_1=1}^{(M,M)} - \frac{1}{4} EU_{p_1 \in (0,1)}^{(M,L)} \right) \right]^{\frac{1}{c-m_1}}$$

To show that there exists a unique $y \in (0, 1)$ such that $[w^*(y)]^{c-m_1} = y^{c-m_1}$ notice that (a) function $s(y) = y^{c-m_1}$ is continuous and strictly increasing on $y \in (0, 1)$ with $s(0) = 0 < 1 = s(1)$, (b) function $g(y) = (w^*(y))^{c-m_1}$ is increasing and convex in $y \in (0, 1)$, (c) $a > a^* \Rightarrow 0 < \lim_{y \rightarrow 0} g(y) < \lim_{y \rightarrow 1} g(y) < 1$. Finally, it can be easily checked that M type prefers to play $plat_1^M = (y, 1 - y)$ rather than separate and play $plat_1^M = (0, 1)$. Thus, if $a > a^*$ and $\max\{m_1, m_2\} < c$ then there exists a *PE* of the dynamic election game in which moderate candidates in the primary pander to the median Democrat by putting weight y on the liberal position, where y is determined as follows

$$y^{c-m_1} = \frac{m_1}{c} \left[\frac{f_0(M-L)}{4} + \frac{f_1}{4} \cdot \frac{(R-M)(y^{m_1} - 2x^{m_2}) + (R-L)x^{m_2}y^{m_1}}{x^{m_2} + y^{m_1}} \right]$$

If $m_1 < c \leq m_2$ then moderate candidates play $plat_1 = (y, 1 - y)$ in the primary, where

$$y^{c-m_1} = \frac{m_1}{c} \cdot \frac{f_1(R-M) + f_0(M-L)}{4}$$

q.e.d.

9.2 Comparative Statics

We detail here the proof of Theorem 3. The proof of Theorem 2 is similar and omitted for brevity.

Proof of Theorem 3. Consider $m_2 < c$ and re-write the system that determines optimal behavior of candidates in *PE* using the following notation: $z = y^{m_1}$, $t = x^{m_2}$, $k = \frac{c}{m_1}$ and $b = \frac{c}{m_2}$:

$$\begin{cases} k \cdot z^{k-1} = \frac{f_0(M-L)}{4} + \frac{f_1}{4} \cdot \frac{(R-M)(z-2t) + (R-L)tz}{t+z} \\ b \cdot t^{b-1} = (R-L)f_1 \cdot \frac{z}{z+t} \end{cases} \quad (5)$$

Then,

$$k \left(\frac{bt^b}{(R-L)f_1 - bt^{b-1}} \right)^{k-1} = \frac{f_0(M-L)}{4} + \frac{tb^{t+1} - 2f_1t(R-M)}{4} + \frac{3bt^b(R-M)}{4(R-L)} \quad (6)$$

Equation (6) has only one unknown - t - and we are interested in $\frac{dt}{dk}$. Recall that we have shown before that for $m_2 < c$ there is a unique solution (t, z) such that $t \in (0, 1)$ and $z \in$

$(0, 1)$ determined by the system above. Therefore, we must have $0 < \frac{bt^b}{(R-L)f_1-bt^{b-1}} < 1$. To see that $\frac{dt}{dk} > 0$, notice that (a) LHS of equation (6) is increasing function of t , (b) RHS of equation (6) does not depend on k , and (c) LHS decreases with an increase in k . Thus, $\frac{dt}{dk} > 0$ which means that $m_1 \uparrow \Rightarrow k \downarrow \Rightarrow x^{m_2} \downarrow$. Finally, we show that when m_1 increases y^{m_1} decreases. Use original equation that determines optimal x :

$$\frac{c}{m_2} x^{c-m_2} = (R-L)f_1 \cdot \frac{y^{m_1}}{y^{m_1} + x^{m_2}}$$

$$x^{c-m_2} > \bar{x}^{c-m_2} \Rightarrow (R-L)f_1 \cdot \frac{y^{m_1}}{y^{m_1} + x^{m_2}} > (R-L)f_1 \cdot \frac{\bar{y}^{m_1+1}}{\bar{y}^{m_1+1} + \bar{x}^{m_2}} \Rightarrow y^{m_1} \bar{x}^{m_2} > \bar{y}^{m_1+1} x^{m_2}$$

where (x, y) is the solution of the system (5) for m_1 and (\bar{x}, \bar{y}) is the solution of the system (5) for $m_1 + 1$. But $\bar{x}^{m_2} < x^{m_2}$ therefore to make sure that last inequality holds true, we must have $y^{m_1} > \bar{y}^{m_1+1}$, which completes the proof that $m_1 \uparrow \Rightarrow y^{m_1} \downarrow$. Similar argument can be applied to the case $m_2 \geq c$, **q.e.d.**

Proof of Corollary 2.

$$Pr[L \text{ wins election}] = \frac{3 - y^{m_1}}{4} \left[f_0 + f_1 x^{m_2} \frac{y^{m_1}}{y^{m_1} + x^{m_2}} \right]$$

$$Pr[M \text{ wins election}] = f_0 \frac{1 + y^{m_1}}{4} + f_1 \frac{x^{m_2}(1 - y^{m_1})^2 + y^{m_1}(1 + y^{m_1})}{4(x^{m_2} + y^{m_1})}$$

$$Pr[\text{Democrat wins}] = f_0 + \frac{1 + y^{m_1}}{4} \cdot f_1$$

The last expression decreases in m_1 , **q.e.d.**

9.3 Welfare Analysis

Proof of Theorem 4

For $m_2 \geq c$,

$$Pr[R \text{ wins}] = 1 - f_0 - \frac{f_1}{4} \cdot (1 + y^{m_1}) \text{ and } Pr[L \text{ wins}] = \frac{3 - y^{m_1}}{4} f_0$$

Thus, $m_1 \uparrow \Rightarrow Pr[R \text{ wins}] \uparrow$ and $Pr[L \text{ wins}] \uparrow$. This implies that $m_1 \uparrow \Rightarrow Pr[M \text{ wins}] \downarrow$ because $Pr[L \text{ wins}] + Pr[M \text{ wins}] + Pr[R \text{ wins}] = 1$.

$$EW^{\text{Dem}} = \left[\frac{3 - y^{m_1}}{4} f_0 \right] \cdot W^{\text{Dem}}(L) + \left[\frac{f_0 + f_1}{4} (1 + y^{m_1}) \right] \cdot W^{\text{Dem}}(M) + \left[1 - f_0 - \frac{f_1}{4} \cdot (1 + y^{m_1}) \right] \cdot W^{\text{Dem}}(R)$$

The result follows immediately after algebraic manipulations with the expression for EW^{Dem} , **q.e.d.**

9.4 Both parties hold simultaneous primaries.

The policy space is $P = [-1, 1]$. Voters with ideal points $z_i \in [-1, 0]$ belong to the Democratic party ($m^{\text{Dem}} = -\frac{1}{2}$) and voters with ideal points $z_i \in [0, 1]$ belong to the Republican party ($m^{\text{Rep}} = \frac{1}{2}$). The location of the median voter in general elections is not known during the primaries: $m^{\text{Pop}} \sim U[-a, a]$ with $a > 0$. Utility of a voter i with ideal point z_i when policy p is implemented is $u(z_i, p) = -|z_i - p|$.

There are two candidates from each party who compete for the nomination. Each Democratic candidate is equally likely to have a true ideological type L or M^D , while each Republican candidate is equally likely to have a true ideological type R or M^R , where

$$-\frac{1}{2} \leq L < M^D \leq 0 \leq M^R < R \leq \frac{1}{2} \text{ where } M^D + M^R = 0 \text{ and } L + R = 0$$

Primaries in both parties occur simultaneously, and we allow the two primaries to have different intensities: m_1^D denotes the intensity of the Democratic primary and m_1^R denotes the intensity of the Republican primary. The winners of the primaries compete with each other in the general election, the intensity of which is m_2 . The candidate with type t receives the utility of $-|t - p|$ if policy p is implemented and pays the cost of lying lie^c where $c \in \mathbb{N}$ if he lies in either stage of the election. We focus on high intensity general election with $m_2 \geq c$.

Given $q_2 = \Pr[t^{\text{Rep}} = M^R]$ (which represents the posterior belief that the Republican nominee has a moderate type after the general election campaign) the probability that the Democratic nominee wins general elections varies with his expected type captures by belief $p_2 = \Pr[t^{\text{Dem}} = M^D]$

$$f(p_2)|_{q_2} = \frac{\frac{Et^{\text{Dem}} + Et^{\text{Rep}}}{2} + a}{2a} = \frac{2a - q_2(R - M^R)}{4a} + \frac{R - M^R}{4a} \cdot p_2 = f(0)|_{q_2} + f_1 \cdot p_2$$

The function $f(p_2)|_{q_2}$ satisfies two intuitive properties: (1) the Democratic candidate that is more likely to have a moderate type has a higher chance to win general elections and (2) the higher q_2 the lower the probability that the Democratic candidate with $p_2 = 0$ wins the general election.¹⁵

Proof of Theorem 5. The condition $m_2 \geq c$ ensures that all types will be separating in the general election stage. The condition $a > \bar{a} = \frac{R+M^R}{2}$ guarantees that in both primaries a candidate with an uncertain type wins over a candidate with definitely moderate

¹⁵This intercept $f(0)|_{q_2}$ works like the location of incumbent R in the basic model - parallel shift of the probability to win generals function. The slope of this function $f_1 = \frac{R-M^R}{4a}$ stays the same for any $q_2 \in [0, 1]$.

type. Now we derive optimal strategy of candidates in the primary elections in each party (using the same technique as in the basic model). Extreme candidates are happy to put all the weight on the extreme position, while moderate types will mimic extreme behavior by lying. The extent of lying depends on the intensities of both primaries (m_1^D, m_1^R) . Denote by x (y) the amount of lying that moderate Democrats (Republicans) are willing to engage in during the primaries, then for any set of parameters $(a, c, m_1^D, m_1^R, M^R, R)$ there exists a unique pair $(x, y) \in (0, 1) \times (0, 1)$ that solves the two equations below:

$$x = \left(\frac{m_1^D}{4c} (A - B) \right)^{\frac{1}{c - m_1^D}} \quad \text{and} \quad y = \left(\frac{m_1^R}{4c} (C - D) \right)^{\frac{1}{c - m_1^R}}$$

where

- A denotes the utility of type $t = M^D$ when type $t = M^D$ wins Democratic primary
- B denotes the utility of type $t = M^D$ when type $t = L$ wins Democratic primary
- C denotes the utility of type $t = M^R$ when type $t = M^R$ wins Republican primary
- D denotes the utility of type $t = M^R$ when type $t = R$ wins Republican primary

The condition $c > \max\{m_1^D, m_1^R\}$ guarantees that moderate candidates prefer to mimic extreme candidates during the primaries rather than separate. This completes the proof that there exists pandering equilibrium described in Theorem 5. In order to show that pandering equilibrium characterized above is unique given the specified conditions, we note that in any other possible configuration of strategies, an extreme type would prefer to deviate and put all the weight on the extreme position, which guarantees that the moderate type will always have an incentive to partially mimic extreme behavior in the primary to increase his chances of winning the nomination, **q.e.d.**

Proof of Corollary 3 is similar to the proof of Theorem 3 and, thus, omitted for brevity.

9.5 Continuous Types of Candidates

Proof of Theorem 6

If $a > \frac{1}{6}$ then a candidate that sends equilibrium message $m = m^{\text{Dem}} = -\frac{1}{2}$ wins primary against a candidate that sends any other message $m' \in (-\frac{1}{2}, 0]$ because $\forall z_i \in [-1, -\frac{1}{2}] \Rightarrow Eu(z_i, m)|_{m=-\frac{1}{2}} \geq Eu(z_i, m')|_{m' \neq -\frac{1}{2}}$, where $Eu(z_i, m)|_{m=-\frac{1}{2}} = z_i + \frac{3f_0 - f_1(1+3R)}{12} - R(1 - f_0)$ and $Eu(z_i, m')|_{m' \neq -\frac{1}{2}} = z_i - R(1 - f_0)$.

Second, we show that no candidate wants to deviate from playing the equilibrium strategy. That means, for all $\alpha \in [-\frac{1}{2}, 0]$ we must have $EU(\alpha, -\frac{1}{2}) \geq EU(\alpha, m')|_{m' \neq -\frac{1}{2}}$:

$$EU\left(\alpha, -\frac{1}{2}\right) = \frac{1}{2} \cdot (f_0 + f_1\alpha) - c \cdot \left|\alpha + \frac{1}{2}\right| = \frac{f_0 + f_1\alpha}{2} - c \cdot \left(\alpha + \frac{1}{2}\right)$$

$$EU(\alpha, m')|_{m' \neq -\frac{1}{2}} = 0 - c \cdot |\alpha - m'| = \begin{cases} -c(\alpha - m') & \text{if } \alpha \geq m' \\ -c(m' - \alpha) & \text{if } \alpha < m' \end{cases}$$

which is true if $c < \frac{f_1}{2} \leq f_0$, **q.e.d.**

Steps of the Proof of Theorem 7

- If $a > \frac{1}{2}$ then a more liberal candidate wins primary over the less liberal candidate
- Type p is indifferent between sending pooling message $m = -\frac{1}{2}$ and separating
- Separating types do not want to mimic each others' strategy
- The conditions $f(0) < c < 2f(-\frac{1}{2})$, $a > \frac{1}{2}$, $f(0) < 1$ and $f(-\frac{1}{2}) > 0$ also guarantee that there exists $p \in (-\frac{1}{2}, 0)$ that satisfies the definition of p
- Candidates with types $\alpha \in (p, 0]$ do not want to deviate.
- Types that pool at $m = m^{\text{Dem}}$ don't want to deviate.

q.e.d.